

# A Formal Proof of Modus Ponens in Lean 4

Agentic NL $\rightarrow$ Lean 4 Pipeline

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## Abstract

We state and verify the inference rule *modus ponens*: if  $P$  implies  $Q$  and  $P$  holds, then  $Q$  holds. The result is formalized in *Lean 4* using *Mathlib* and discharged by direct application. The verification confirms that the foundational rule of detachment is provable from the propositional primitives of Lean’s logic without auxiliary axioms. The takeaway is a minimal, fully checked anchor for downstream propositional reasoning in the pipeline.

## 1 Introduction

Modus ponens is the rule of detachment underlying nearly every deductive system since antiquity. It licenses the passage from a conditional and its antecedent to its consequent. Although elementary, it provides the smallest non-trivial test of a proof assistant’s handling of implication. We record a machine-verified proof of the following statement.

**Theorem 1** (Modus Ponens). *For all propositions  $P$  and  $Q$ , if  $P \Rightarrow Q$  and  $P$ , then  $Q$ .*

## 2 Formal Statement

The Lean 4 theorem header is the following.

```
theorem modus_ponens (P Q : Prop) (hpq : P -> Q) (hp : P) : Q
```

## 3 Natural Language Proof

Let  $P$  and  $Q$  be propositions. Assume  $P \Rightarrow Q$  and assume  $P$ . By hypothesis,  $P \Rightarrow Q$  asserts that whenever  $P$  holds, so does  $Q$ . Since  $P$  holds, applying the implication to this fact yields  $Q$ . Therefore  $Q$  holds.  $\square$

## 4 Formal Lean 4 Proof

The Lean 4 proof is the direct application of the implication to its antecedent.

```
theorem modus_ponens (P Q : Prop) (hpq : P -> Q) (hp : P) : Q
  := hpq hp
```

The proof uses no Mathlib lemmas or tactics; it relies solely on the native function-application rule of Lean 4's core logic.

## 5 Conclusion

We have machine-verified modus ponens in Lean 4 with Mathlib. The proof closes in a single step, providing a trustworthy primitive for larger formal developments produced by the pipeline.